ABDUCTION IN GENERATING CONJECTURES IN DYNAMIC GEOMETRY THROUGH *MAINTAINING DRAGGING*

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This paper introduces two types of abduction associated to two different ways of generating conjectures that arise from using a particular dragging modality in dynamic geometry. We refer to such dragging modality as maintaining dragging (MD). A first use of MD relies on physical dragging-support and it seems to lead the solver "automatically" to the formulation of a conjecture. In this case the abductive reasoning seems to occur at a meta-level and to be concealed within the MD-instrument. On the other hand, some of our data shed light onto a different way of generating conjectures which is rooted in use of MD but is "freed" from the physical dragging-support. In this case abductive reasoning seems to occur at the level of the dynamic exploration.

key words: abduction, conjecture-generation, dragging, dynamic geometry, instrument, instrumented abduction, maintaining dragging (MD), psychological tool

INTRODUCTION

This paper reports on some findings of a study on conjecture-generation in a dynamic geometry system (DGS). The study[1] blossomed from results of Italian research that provided a classification of various dragging modalities used by solvers during explorations of open problem activities (Arzarello et al., 2002). This classification describes different ways of dragging points on the screen as a conjecture is elaborated and tested, and the solver's control shifts from "ascending" to "descending" (Arzarello et al., 1998). Such transition was described as occurring in correspondence to use of *dummy locus dragging*, that is moving a basic point so that the drawing *keeps* a discovered property, and to be promoted by abduction (Arzarello et al., 2002). Our study aimed at unravelling the relationship between abduction and the use of particular dragging modalities.

Since according to the literature (Olivero, 2002), spontaneous use of dummy locus dragging does not seem to occur frequently, first, we explicitly introduced the participants of the study to four dragging modalities, elaborated from Arzarello et al.'s classification, during two in-class lessons. The modality elaborated from *dummy locus dragging* that we introduced is what we refer to as *maintaining dragging* (MD), and it consists in dragging a base-point of the dynamic figure on the screen trying to maintain some geometrical property of the figure. In other words, performing MD consists in identifying a property that the figure can have and trying to induce such property as a *soft*[3] *invariant* during dragging.

In order to unravel the relationship between abduction and the use of MD during a process of conjecture-generation, we constructed a model (Baccaglini-Frank, 2010;

Baccaglini-Frank & Mariotti, 2010) that provides a cognitive description of the process, as a sequence of (implicit) tasks that the solver seems to address. During the study we tested, refined, and tested again the model, which was used as a tool of analysis in the final round of data analysis. We proceeded through two rounds of 90-minute clinical interviews[2] with pairs of students, each round with different students who had participated to the introductory lessons on dragging. Data was collected in the form of screenshots with audio, video recording, students' Cabri files and work on paper, transcriptions and subtitled videos. The students (a total of 31), between the ages of 15 and 18, were from three Italian high schools and had been using Cabri in the classroom for at least one year prior to their interviews.

A few theoretical background notions

The study made use of the notions of *abduction* and *instrument* as follows.

Abduction. Peirce was the first to introduce the notion of abduction as follows:

> ...abduction looks at facts and looks for a theory to explain them, but it can only say a "might be", because it has a probabilistic nature. The general form of an abduction is: a fact A is observed; if C was true, then A would certainly be true; so, it is reasonable to assume C is true (Peirce, 1960, p. 372).

Recently, there has been renewed interest in the concept of abduction, with a number of studies focused on its various uses in mathematics education (see for example Baccaglini-Frank, 2010, chapter 2). For this paper we will refer to the definition introduced above and to Magnani's description of abduction as an *explanatory hypothesis* (2001, pp. 17-18).

Instrument. The study considers "dragging" in a DGS after the instrumentation approach (Vérillon & Rabardel, 1995; Rabardel & Samurçay, 2001; Rabardel, 2002), as has been done fruitfully by other researchers (for example, Lopez-Real & Leung, 2006; Leung, 2008; Strässer, 2009). A particular way of dragging, in our case MD, may be considered an *artifact* that can be used to solve a particular *task* (in our case that of formulating a conjecture). When the user has developed particular *utilization schemes* for the artifact, we say that it has become an *instrument* for the user. We will call the utilization schemes developed by the user in relation to particular ways of dragging, "dragging schemes". In this sense the model we developed can be interpreted as the description of a utilization scheme for MD, with respect to the task of generating a conjecture. From now on we will refer to our model as the MD-conjecturing model.

INSTRUMENTED ABDUCTION

The successive analyses of our data led to the development of a new notion, that of *instrumented abduction*, through which we describe the place and role of abduction in the process of conjecture-generation we have studied (Baccaglini-Frank & Mariotti, 2010; Baccaglini-Frank, 2010). We introduce this notion by providing excerpts of two solvers' exploration that constitute a paradigmatic example of how MD can be used in the process of conjecture-generation and how the MD-conjecturing model can be used as a tool of analysis.

James and Simon were given the following open-problem activity:

> Construct three points *A*, *B*, and *C* on the screen, the line through *A* and *B*, and the line through *A* and *C*. Then construct the parallel line *l* to *AB* through *C*, and the perpendicular line to *l* through *B*. Call the point of intersection of these last two lines *D*. Consider the quadrilateral *ABCD*. Make conjectures on the kinds of quadrilaterals can it become, trying to describe all the ways it can become a particular kind of quadrilateral.

The solvers followed the steps that led to the construction of *ABCD* (Fig. 1) and soon noticed that it could become a rectangle. Simon was holding the mouse (as shown by his name being in bold letters in the excerpts below), and followed James' suggestion to use MD to "see what happens" when trying to maintain the property *ABCD rectangle* while dragging the base-point *A*. The solvers have accomplished Task 1 of our model (Baccaglini-Frank & Mariotti, 2010): determining a configuration to be explored by inducing it as a (soft) invariant. In such situation we refer to the selected property *ABCD rectangle*, as *intentionally induced invariant*. As Simon was focused on performing MD, James' attention seemed to shift to the movement of the dragged-base-point, and he proposed to "do trace" in order to "see if …[*A* moves along a "pretty precise curve"]." James seemed to be looking for something that *A* can be dragged along in order for *ABCD* to remain a rectangle, thus addressing Task 2 of our model (searching for a condition, through MD, that makes the intentionally induced invariant be visually verified, and recognizing a condition in the movement of dragged-base-point along a *path*). This intention seems to indicate that James has conceived an object along which dragging the base point *A* will guarantee that the intentionally induced invariant is visually verified. This is what we call a *path*. Moreover he is trying to "understand" what such path might be. In other words he is searching for a *geometric description of the path*. To do this he suggests activating the trace on *A* as Simon performs MD (Fig. 2).

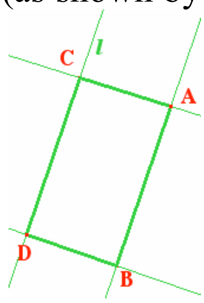

Figure 1

Excerpt 1

1	I:	and you, James what are you looking at?
2	James:	That it seems to be a circle...
3	**Simon**:	I'm not sure if it is a circle...
4	James:	It's an arc of a circle, I think the curvature suggests that.
…		
10	James:	Ok, do half and then more or less you understand it, where it goes through.
11	**Simon**:	But *C* is staying there, so it could be that *BC* is...is
12	James:	right! because considering *BC* a diameter of a circle…

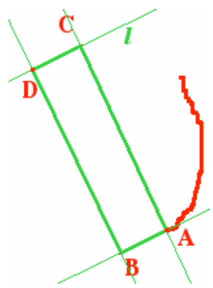

Figure 2

[They construct the circle and drag *A* along it, and then they write the conjecture: "*ABCD* is a rectangle when *A* is on the circle with diameter *BC*."]

In this Excerpt James identifies a regularity in the movement of the dragged-base-point as "a circle" ([2], [4]) "considering *BC* a diameter" ([12]). Moreover, there seems to be the intention of looking for something, which we interpret as an attempt at "making the path explicit." This can lead to perceiving a second invariant, that we call the *invariant observed during dragging*, as a regular movement of the dragged-base-point. Both invariants are perceived within the phenomenological domain of the DGS, where a relationship of "causality" may also be perceived between them. Of course such relationship can be formulated within the domain of Euclidean Geometry as a *conditional link* between geometrical properties corresponding to the invariants, provided that the solver gives an appropriate geometrical interpretation. This can be expressed through a conjecture and checked (Task 3: checking the conditional link between the invariants and verifying it through the dragging test).

Solvers like Simon and James who use MD effectively for generating a conjecture seem to withhold the key for making sense of their findings. This seems to consist in conceiving, within the phenomenology of the DGS, the invariant observed during dragging as a "cause" of the intentionally induced invariant, and then, within the domain of Euclidean geometry, in interpreting such cause as a geometrical "condition" for the intentionally induced invariant, a geometrical property of the figure, to be verified. In other words, the solvers establish a causal relationship between the two invariants generating – as Magnani says (2001) – an *explanatory hypothesis* for the observed phenomenon.

From the data analyzed another characteristic of behaviors like that of Simon and James is the use of MD in an "automatic" way. That is, the solver proceeds through steps that lead *smoothly* to the discovery of invariants and consequently to the generation of a conjecture, with *no apparent abductive ruptures* in the process. So where is abduction when conjecture-generation occurs as described by the MD-conjecturing model? Abduction can be recognized in the expert's interpretation of the invariant observed during dragging as a cause for the intentionally induced invariant to be visually verified. Thus, automatic use of MD does not seem to produce explicit abductive arguments during the exploration leading to a conjecture; instead it seems to *condense and subsume* the abductive process. We introduce the new notion of *instrumented abduction* to refer to an abductive inference *supported by* an instrument like in this case. Here the instrument is the combination of MD (artifact) with the MD scheme (utilization scheme) described in the MD-conjecturing model.

USE OF MAINTAING DRAGGING AS A PSYCHOLOGICAL TOOL

Our study was primarily aimed at developing and subsequently testing the MD-conjecturing model. Our final data analyses seemed to confirm the model, however one case opened a window onto a fundamentally different way of generating a conjecture that seems to have roots in use of MD even though no dragging is actually performed. Below we summarize the exploration in which we found such evidence and present an excerpt from it. The solvers, Francesco and Gianni, were given the following task:

> Draw a point *P* and a line *r* through *P*. Construct the perpendicular line *l* to *r* through *P*, construct a point *C* on it, and construct the circle with center in *C* and radius *CP*. Construct the symmetric point of *C* with respect to *P* and call it *A*. Draw a point *D* on the semi-plane defined by *r* that contains *A*, and construct the line through *D* and *P*. Let *B* be the second intersection with the circle and the line through *P* and *D*. Consider the quadrilateral *ABCD*. Make conjectures on the kinds of quadrilaterals can it become, trying to describe all the ways it can become a particular kind of quadrilateral.

Francesco and Gianni had effectively used MD to generate conjectures in previous explorations. In this particular exploration they had noticed the potential property *ABCD parallelogram*. Thus Francesco had chosen a base-point to drag while trying to maintain such property. However Francesco and Gianni seemed to conceive a geometric description of the path that did not coincide with their interpretation of the trace mark left on the screen as Francesco performed MD. This led the solvers to reject the original description and search for a new condition for maintaining *ABCD parallelogram*. The solvers were not able to reach such condition using MD and they interrupted all forms of dragging. After a moment of silence Gianni started speaking about constructing a circle to drag along, as shown in the following excerpt.

Excerpt 2

1	Gianni:	eh, since this is a chord, it's a chord right? We have to, it means that this has to be an equal chord of another circle, in my opinion with center in *A*. because I think if you do, like, a circle with center
2	Francesco:	*A*, you say…
3	Gianni:	symmetric with respect to this one, you have to make it with center *A*.
4	Francesco:	uh huh
5	Gianni:	Do it!
6	Francesco:	with center *A* and radius *AP*?
7	Gianni:	with center *A* and radius *AP*. I, I think…

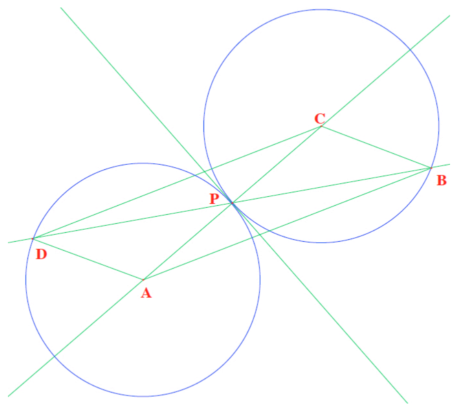

Figure 3

Gianni appears to be trying to solve the problem of finding a way to drag *D* in order to maintain the property *ABCD parallelogram* as if he had to perform MD. However the solvers' inability to perform MD successfully, led to the argumentation above in which the following abductive inference (in Pierce's terms) is evident:

- fact: *DP*=*PB* (recognized as chords [1])

- rule: given symmetric circles with *DP* and *PB* symmetric chords ([1], [3]), then *DP=PB* (as observed)
- abductive hypothesis: there exists a symmetric circle with center in *A* and radius *AP* ([3]-[7]).

Without further hesitation the solvers formulate their conjecture (Fig.3): "*D* belongs to the circle centered in *A* with radius *AP* implies *ABCD* parallelogram."

We highlight how Gianni applies a way of reasoning, that has roots in his knowledge of the MD scheme, to a substantially different situation. Gianni is trying to find a condition for *ABCD* to be a parallelogram, but instead of focusing on the movement of a *point* (*D*) as would have occurred during use of MD, Gianni notices *chords* (*BP* and *PD*, which he interprets as a chord) and visualizes their symmetric behavior, which leads him to produce an *explicit* abductive argument. In particular now the "rule" *appears*, while in the case of instrumented abduction such rule would have remained implicit in the movement of the dragged base-point and/or the trace mark on the screen.

Taking a Vygotskian perspective (Vygotsky, 1978, p. 52 ff.), the process that was external, supported by the MD-instrument in the case of instrumented abduction, now can be seen as "transformed" into an *internal* process, the way of reasoning of abductive nature that we described above. We can say that the MD-instrument has been *internalized* and it can now be used as a *psychological tool* (Kozulin, 1998) to solve a conjecture-generation problem. Moreover, now we can underline how the intention of searching for a cause that solvers who have appropriated the MD scheme exhibit, resides at a meta-cognitive level (Gollwitzer & Schaal, 1998) with respect to each specific investigation the solvers engage in. Thus, instrumented abduction resides at such meta-cognitive level, while the abductive inference in the second case resides at the level of the dynamic exploration.

A HYPOTHESIS ON PROOF

If we consider conjectures generated in the two different ways described above, the differences between them are not in the *product* of the dynamic exploration, the statement of the conjecture, but in the elements that emerge during the *process* of the exploration. When MD is used "automatically" as in the conjecture-generation process characterized by instrumented abduction, the premise and the conclusion of the statement of the final conjecture seem to be "distant". That is, these conjectures seem to exhibit a "gap" between the premise and the conclusion, because no bridging arguments tend to emerge from the exploration. On the other hand, it seems that when MD is internalized and used as a psychological tool, the produced conjectures are accompanied by arguments that can be used to bridge the premise and the conclusion. In other words, in this case the final conjectures seem to exhibit less of a "gap" between premise and conclusion. Below we illustrate an episode that provides evidence supporting such a hypothesis.

After they had reached their conjecture (Excerpt 2), Francesco and Gianni produced the following oral proof.

Excerpt 3

Francesco: ah, no! but wait! we know a lot of things here, excuse me, if *DA* is equal to *AP* which is equal to *PC* which is equal to *CB*, *DAP* and *PCB* are isosceles.

Gianni: yes… And so the angles, right!

Francesco: Wait, and so this [pointing to the angle *ADP*]…

Gianni: the angles over there and down there are..

Francesco: so, let's say *ADP* is equal to *APD*, which is equal to

Gianni: we know that these, these are also opposite at the vertex and so they are *all* equal those angles there.

…

Francesco: but, excuse me, if this… if the angles at the base, are equal, also, obviously, the angle at the vertex, uhm, the angle *DAP* is equal to *PCB* necessarily because of the sum of angles.

Gianni: Yes, right.

Francesco: Because it is 180° minus equal angles

Gianni: okay, so this way we understood that the two triangles are equal.

Francesco: Exactly.

Gianni: And so also *PD* and *PB* are equal.

Francesco: Okay, so the diagonals divide each other in their midpoints, and therefore *ABCD* is a parallelogram.

Gianni: Yes, right. [Smiling]

In the analysis of the Excerpt 2 we described how Gianni focuses on the two segments *PB* and *PD*, and interprets them as chords of symmetric circles. This constitutes the key idea (Raman, 2003) in their oral proof summarized as follows:

- the circles are symmetric so *AD* is congruent to *AP* which is congruent to *PD* and to therefore to *BC*;
- the isosceles triangles *APD* and *PBC* are congruent because they have congruent angles, since the angle *DPA* is opposite at its vertex to *CPB*;
- therefore *PD* is congruent to *PB*,
- so *ABCD* has diagonals that intersect at their midpoints and therefore it is a parallelogram.

The geometrical properties that emerged during the production of the conjecture, now become fundamental ingredients of the solvers' proof. In other words, these geometrical properties seem to help bridge the gap between premise and conclusion of the conjecture. At this point, if we consider conjectures as both the *product* (the statement of the conjecture) and the *process* (the exploration leading to the statement of the conjecture), we can characterize conjectures as those *with a gap* that emerge through automatic use of MD as opposed to those *with bridging elements* that emerge as a product of an internalization of MD. This characterization helps express our hypothesis as follows.

Hypothesis on proof. Automatic use of MD seems to generate conjectures with a gap, while use of MD as a psychological tool seems to generate conjectures with bridging elements. Therefore use of MD as a psychological tool may foster the solver's construction of a proof of the statement of his/her conjecture.

CONCLUDING REMARKS

Through our study we were able to identify two distinct forms of abductive reasoning related to two different ways of generating conjectures that arise from using a particular dragging modality in dynamic geometry. When MD is used automatically through physical dragging, the abductive reasoning seems to reside at a meta-level with respect to the dynamic exploration. This idea is condensed in the notion of instrumented abduction that we introduced. On the other hand, when MD seems to be "freed" from the physical support, and internalized, the abduction seems to occur at the level of the exploration. In this case the conjecture-generation process seems to have the advantage of involving arguments that can be reinvested in a successive proof, like in the case of Francesco and Gianni.

We hypothesize that conjectures generated "automatically" through physical use of MD, that is *conjectures with a gap*, will present *cognitive rupture* with respect to a potential proof since the solver will have no arguments emerging from the conjecturing-process to base his/her proof upon. This seems to be the case because the process of conjecture-generation is supported by the DGS and mostly concealed within it, as is the abductive inference that we refer to as *instrumented abduction*. On the other hand, we hypothesize that if solvers who have appropriated the MD-instrument *also internalize it* transforming it into a psychological tool, or a fruitful "mathematical habit of mind" (Cuoco, 2008) that may be exploited in various mathematical explorations leading to the generation of conjectures, a greater cognitive unity (Pedemonte, 2007) might be fostered. In other words, it may be the case that when the MD-instrument is used as a psychological tool the conjecturing phase is characterized by the emergence of arguments that the solver can set in chain in a deductive way when constructing a proof (Boero et al., 1996). In particular we think this may occur if, as in the case of Francesco and Gianni, abduction in which the rules are taken from the domain of the Theory of Euclidean Geometry is used during the process of conjecture-generation. An abduction of this sort seems to expose key ideas that can be reinvested in the proof.

The relatively small amount of data analyzed in our study does not allow us to make general statements about the hypothesis on proof we illustrated above. Moreover our study was not focused on investigating internalization of the MD-instrument and its transformation into a psychological tool: the case of Gianni and Francesco was an unexpected isolated instance that suggested new potential insight into how a DGS can be used (or not) in the context of argumentation and proof, opening an alley for future research. For example, as some colleagues have suggested, it would be interesting to investigate *what it takes*, both from learning and teaching perspectives, for the solver to make the cognitive shift we describe, transforming the MD-instrument into a psychological tool.

NOTES

1. Research study partly funded by PRIN 2007B2M4EK (Instruments and representations in the teaching and learning of mathematics: theory and practice).

2. The activities proposed were open-ended tasks. The interviewer would typically ask the solver to explain an action, to describe what s/he was looking at or trying to accomplish, or to provide clarification or elaboration of a statement s/he made.

3. We use the terminology "soft" and "robust" as introduced by Healy (2000).

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